

# Robust Navigable Cores in the Human Brain Networks

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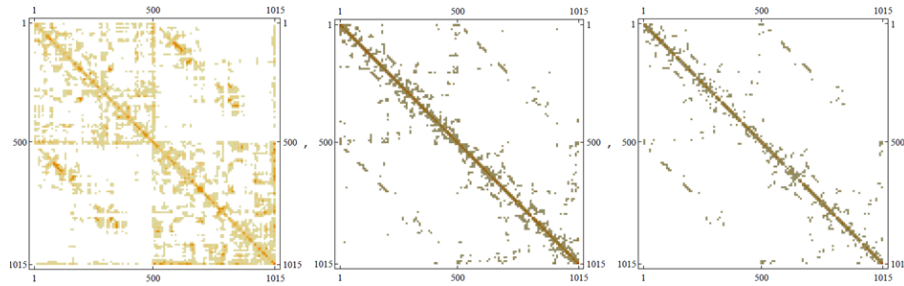
## 1 Introduction

Given a set of network nodes with coordinates in an arbitrary metric space, the greedy navigational core (GNC) can be identified and calculated, as the minimum set of links between these nodes which provides 100% greedy navigability [2]. The level of presence of the Greedy Navigational Core is high in various real networks, and can also be disclosed in structural networks of the Human Brain. In [2] we have shown that the GNC precision (the ratio of the number of the GNC links included in the real network and the total number of GNC links, sometimes referred to as true positive rate) is 89% in a five-subject based averaged structural brain network.

## 2 Results

Here we perform results on a follow-up investigation of structural greedy navigability in 200 structural brain networks from 40 individual subjects at 5 different scales (these scales correspond to resolutions of 83, 129, 233, 463, 1015 nodes in the brain structural networks) [3]. For the GNC network generations only the physical (3D Euclidean) coordinates of the brain parcels were used, no other anatomical data or considerations were utilized. We found that the level of GNC precision (the number of GNC links in the brain network) is quite consistent among the 40 brain networks within all scale, in spite of the fact that the 40 brain networks significantly differ from each other. The mean (and the standard deviation) of the GNC precisions in different scales (with increasing resolutions) are 0.85 (0.025), 0.88 (0.019), 0.81 (0.017), 0.70 (0.017), 0.51 (0.025), respectively. GNC precisions turned out to be also robust against weak (possibly spurious) connections. For instance, when 50% of the links are removed from the brain networks the precisions remain close to the original values: 0.72 (0.030), 0.75 (0.019), 0.68 (0.020), 0.57 (0.019) 0.42 (0.020). One can also observe that larger part of the navigational core is missing from higher resolution brain networks, however, the precisions are still consistent.

In scale 5 we have constructed a sequence of average networks, based on averaging networks over link weights (inferred from measured anatomical strengths of fiber paths), and cutting out links with small average weights. We have also generated an



**Fig. 1.** Plots of the adjacency matrix of a 1015 node structural brain network, the greedy navigational core (GNC) network, and that part of the GNC which is included in the brain network.

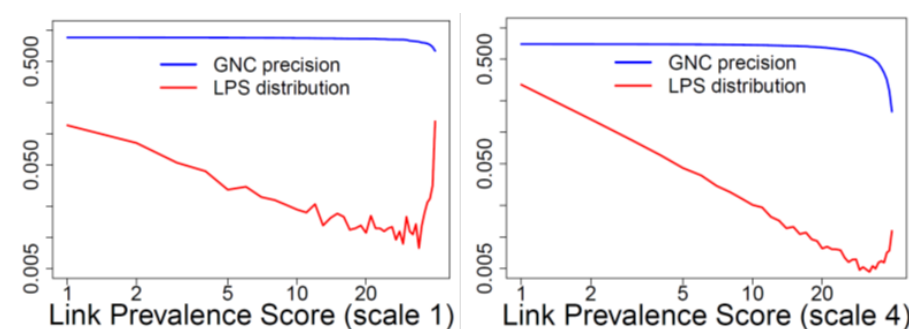
average GNC network based on averaging the centroids of the brain parcels. The size of the average GNC network (the number of links) is 2652. Regarding the average network sequence, the following table shows the GNC precision in the function of the size of the average network:

Threshold	$10^{-8}$	$10^{-7}$	$10^{-6}$	$10^{-5}$	$10^{-4}$	$10^{-3}$	$10^{-2}$
Averaged Brain network size	101468	101450	95929	67553	39406	15996	2381
GNC Precision	0.925	0.925	0.924	0.907	0.874	0.769	0.263

Note, that if none of the links have been cut out, the average brain network contains 101468 links (full mesh would have 514 605 links) and in this case 0.923 fraction of the 2652 links of the average GNC are in this network. If the number of links in the average network is quite comparable to the sizes of individual networks (Threshold=0.001, size of average network=15996), the GNC precision is still as high as 76.8%. When the size of the average network is comparable to the average GNC network, the GNC precision is still amazingly 26.28%.

Besides anatomical weighting strategies, the link prevalence score (the number of networks containing the link) can also be used to identify possibly existent (high prevalence score) and non-existent (low prevalence score) links in the inferred brain structural networks, and based on this one can compromise the false positives and false negatives in pruning the networks [1]. Here we also present results on GNC precisions in individual networks (the above mentioned 40-subject-5-scale networks with the number of nodes 83, 129, 234, 463, 1015 in scale 1, 2, 3, 4, 5, respectively) thresholded by the link prevalence scores. Thresholding means in this case that in an individual network only that links are kept, which are present at least  $T - 1$  other networks too, where  $T$  is the threshold. Within a resolution, every network is thresholded by all possible values of LPS's (1,...,40), then the GNC precisions are measured in all resulted networks. The GNC precisions are then averaged over the subjects for every LPS threshold. The important observation is that GNC precisions are consistent (low variations across subjects) and robust against LPS thresholding in all scale (see Figure 1). For example in scale 1 for LPS=1 (no link is removed) GNC precision is 0.85 and for LPS=30 (about 30% of links are removed from every network) GNC precision is still as high as 0.80. In all the 5 scales it can be observed that for lower values of LPS thresholds the GNC

precision remains almost intact while for higher values its decrease is fastening. The fastening decrease measurably coincides with the right hand side (consisting of possibly existing links) of the link prevalence distribution. This means that most of the true positive links in GNC networks are also possibly existent. The consistency and robustness of the greedy navigational core network precisions are remarkable in the light that these networks are inferred only from the physical coordinates of the brain parcels by straightforward geometric computations. Based on these we think navigational cores and their precisions are possible candidates for auxiliary testing and qualifying sets of structural networks of the Human Brain.



**Fig. 2.** Greedy Navigational Core precision and Link Prevalence Score distribution in the function of Link Prevalence Score

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